



## Robust output feedback sampling control based on second order sliding mode

Franck Plestan, Emmanuel Moulay, Alain Glumineau, Thibault Cheviron

### ► To cite this version:

Franck Plestan, Emmanuel Moulay, Alain Glumineau, Thibault Cheviron. Robust output feedback sampling control based on second order sliding mode. *Automatica*, 2010, 46 (6), pp.1096-1100. 10.1016/j.automatica.2010.03.004 . hal-00480076

**HAL Id: hal-00480076**

**<https://hal.science/hal-00480076>**

Submitted on 3 May 2010

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Robust output feedback sampling control based on second order sliding mode

Franck Plestan <sup>a</sup>, Emmanuel Moulay <sup>b</sup>, Alain Glumineau <sup>a</sup>, Thibault Cheviron <sup>c</sup>

<sup>a</sup>*IRCCyN, UMR CNRS 6597, Ecole Centrale de Nantes, Nantes, France*

<sup>b</sup>*Xlim-SIC, UMR CNRS 6172, Université de Poitiers, Poitiers, France*

<sup>c</sup>*Délégation Générale de l'Armement DGA, Brest, France*

---

## Abstract

This paper proposes a new second order sliding mode output feedback controller. This latter is developed in the case of finite sampling frequency and is using only output information in order to ensure desired trajectory tracking with high accuracy in a finite time in spite of uncertainties and perturbations. This new strategy is evaluated in simulations on an academic example.

*Key words:* Second order sliding mode, output feedback, sampling controller, robustness.

---

## 1 Introduction

High order sliding mode control is a nonlinear control strategy with property of robustness with respect to uncertainties and perturbation. Several algorithms have been published, more or less usable on practical applications (Levant, 2001; Bartolini *et al.*, 2000; Laghrouche *et al.*, 2006b; Laghrouche *et al.*, 2007; Plestan *et al.*, 2008a). An other property of this class of controller is the finite time stabilization of the controlled system (Moulay and Perruquetti, 2005; Moulay and Perruquetti, 2006). Since few years, applications to experimental set-ups have proved feasibility and applicability of these approaches for robots (high-order sliding mode controllers and observers) (Laghrouche *et al.*, 2006a), electrical machines (Laghrouche *et al.*, 2006a), pneumatic actuators (Laghrouche *et al.*, 2006b; Girin *et al.*, 2009). However, a lack of higher order sliding mode control is the use of sliding variables high order time derivatives. By a practical point-of-view, it can decrease the interest of such controllers, due to the bad effect of measurement noise on the control. In order to remove this lack, a mean is to consider output feedback. The objective is then to propose output feedback control which ensures robustness, finite convergence and accuracy by reducing number and

order of sliding variables time derivatives. Two kinds of approaches are possible. The first one consists in designing state observer or differentiators (Levant, 2007) coupled to a controller: it implies to verify the stability of the observer/differentiators-based controlled system which is in main cases a hard task (in the case of high order sliding mode, it has been done in (Levant, 2003)). The second one consists in using static output feedback. Very few results are available on second order sliding mode static output feedback. In (Bartolini *et al.*, 2000), an optimal version of the so-called “twisting” algorithm has been provided, its main drawback being the requirement of the output derivative sign. An other major result is the so-called “super-twisting” (Levant, 1993) algorithm which requires no information on the output time derivative; however, this controller has been developed for systems with relative degree equal to 1 with respect to the control input. In (Khan *et al.*, 2003), a second order sliding mode output feedback controller is proposed for systems with relative degree equal to 1 or 2: its main drawback is the absence of a formal closed-loop system stability proof. Finally, a first attempt for sampling controller has been proposed in (Plestan *et al.*, 2008b), but this controller also requires output derivative sign. Note that previous works (Levant, 1993; Levant, 2005) have been made for high order sliding mode control (with state feedback) by considering discrete measurements. The current paper proposes a new design strategy for sampling output feedback controller. The interest of such controllers is essentially due to practical considerations:

---

\* Corresponding author F. Plestan. Tel. +33(0)240376914. Fax +33(0)240376930.

Email address: [Franck.Plestan@irccyn.ec-nantes.fr](mailto:Franck.Plestan@irccyn.ec-nantes.fr) (Franck Plestan).

in fact, application of control law to real setup requires finite sampling period, which can fundamentally change the closed loop system behaviour (Plestan *et al.*, 2008b). According to an adequate gain tuning, the proposed method ensures the establishment of a “real” high-order sliding mode (Levant, 1993) in a finite time. Furthermore, whereas existing output feedback controllers (for example “super-twisting” algorithm) are applicable only to 1-relative degree systems, the proposed approach can be applied to a larger class of systems, *i.e.* 1- or 2-relative degree systems. The proposed solution can be also seen in the context of relay controllers (Anosov, 1959; Fridman and Levant, 1996; Fridman and Levant, 2002) and their stability.

The paper is organized as follows. Section 2 states the problem of high order sliding mode output feedback controller. In Section 3, the control strategy is developed. In Section 4, the control solution is applied to an academic example.

## 2 Problem statement

Consider a single-input nonlinear system

$$\dot{x} = f(x) + g(x)u, \quad y = h(x) \quad (1)$$

with  $x \in \mathbb{R}^n$  the state variable,  $u \in \mathbb{R}$  the input, and  $y \in \mathbb{R}$  a smooth output function.  $f$  and  $g$  are smooth uncertain functions. Let  $s(x, t)$  denote the sliding variable defined as

$$s(x, t) = h(x) - h_d(t) \quad (2)$$

$h_d(t)$  being the desired trajectory and a bounded smooth function. Assume that

**H1.** The relative degree of (1)-(2) with respect to  $s$  is constant and equal to two<sup>1</sup>, and the associated zero dynamics are stable. Only the sliding variable  $s$  is measured. ■

The output  $s$  fulfills

$$\ddot{s} = \bar{a}(x) + b(x)u - \ddot{h}_d(t) = a(x, t) + b(x)u \quad (3)$$

Assume that

**H2.** The solutions are understood in the Filippov sense (Filippov, 1988), and system trajectories are supposed to be infinitely extendible in time for any bounded Lebesgue measurable input. ■

**H3.** Functions  $a(x, t)$  and  $b(x)$  are bounded uncertain functions. Without loss of generality, one supposes that

<sup>1</sup> Note that results given in the sequel are also applicable on nonlinear systems with relative degree equal to 1: in this case, the discontinuous control law developed in the sequel is applied on the first time derivative of  $u$ .

$b(x)$  sign is strictly positive. Thus, there exists positive constants  $a_M \geq 0$ ,  $b_m > 0$  and  $b_M > 0$  such that

$$|a(x, t)| \leq a_M, \quad 0 < b_m < b(x) < b_M$$

for  $x \in \mathcal{X} \subset \mathbb{R}^n$ ,  $\mathcal{X}$  being a bounded open subset of  $\mathbb{R}^n$  within which the boundedness of the system dynamics is ensured, and  $t > 0$ . ■

By defining  $z_1 = s$  and  $z_2 = \dot{s}$ , the second order sliding mode output feedback control of (1) with respect to the sliding variable  $s$  is equivalent to the finite time stabilization of the system

$$\dot{z}_1 = z_2, \quad \dot{z}_2 = a + b \cdot u(z_1) \quad (4)$$

under Assumptions H1-H3.

**Definition 1 (Levant, 1993)** *Given the sliding variable  $s(x, t)$ , the “real second order sliding set” associated to (1) is defined as*

$$\mathcal{S}_{T_e} = \{x \in \mathcal{X} \mid |s| \leq k_1 T_e^2, |\dot{s}| \leq k_2 T_e\}. \quad (5)$$

with  $T_e > 0$  the finite sampling time and  $k_1, k_2$  positive constants. ■

**Definition 2 (Levant, 1993)** *Consider the not-empty real second order sliding set  $\mathcal{S}_{T_e}$  (5), and assume that it is locally an integral set in the Filippov sense. The corresponding behavior of system (1) satisfying (5) is called “real second order sliding mode” w.r.t.  $s(x, t)$ . ■*

## 3 A second order sliding mode output feedback controller

In this section, output feedback controllers are proposed in case of finite sampling frequency. In a sake of clarity, the first part of this section is devoted to the design of a controller for a double integrator. Then, the result will be extended to uncertain nonlinear systems (1).

### 3.1 A solution for a double integrator

Consider the following system

$$\dot{z}_1 = z_2, \quad \dot{z}_2 = u \quad (6)$$

with

$$u = -K(t) \operatorname{sign}(z_1(kT_e)) \quad (7)$$

and  $K > 0$  and  $k \in \mathbb{N}$  ( $k$  can be viewed as a time counter). Gain  $K$  is constant on the time interval  $t \in [k \cdot T_e, (k+1) \cdot T_e[$ , and  $k(0) = 0$ .

**Theorem 1** Consider system (6) controlled by (7) and a gain  $K_m > 0$ . Then, there always exists a sufficiently large gain  $K_M$  with  $0 < K_m < K_M < \infty$  such that the gain  $K(t)$  defined as

$$K(t) = \begin{cases} K_m & \text{if } t \notin \mathcal{T} \\ K_M & \text{if } t \in \mathcal{T} \end{cases} \quad (8)$$

with  $\mathcal{T} = \{t \mid \text{sign}(z_1(kT_e)) \neq \text{sign}(z_1((k-1)T_e))\}$  and the control law (7) ensure the establishment of a real second order sliding mode for system (6) with respect to  $z_1$ , i.e. there exists a finite time  $t_F$  such that, for  $t \geq t_F$ ,

$$\begin{aligned} |z_1| &\leq \left[ (K_M - K_m) + \frac{(K_M + K_m)^2}{2K_M} \right] \cdot T_e^2, \\ |z_2| &\leq \frac{K_M + K_m}{2} \cdot T_e \end{aligned} \quad (9)$$

■

**Discussion.** Theorem 1 displays the control law design through the existence of a positive gain  $K_m$  and a sufficiently large gain  $K_M$ . The single condition on  $K_m$  is its positivity. On an other hand, there **always** exists a constant value  $K_{M0}$  depending on sampling period, initial conditions such that, if  $K_M > K_{M0}$ , a second order sliding mode is established.  $K_{M0}$  is the minimal value of  $K_M$  allowing to reach trajectories converging to a vicinity of the origin.

**Proof.** The convergence analysis of system (6) controlled by (7) is made in two parts. The first part proves the convergence to a closer vicinity of the origin at each gain commutation whereas the second step characterizes the stable limit cycle that system trajectories reach in a finite time.

**First Part.** For a sake of clarity, consider Figure 1-Left; the black points on the curves have the coordinates  $z_1(kT_e)$  and  $z_2(kT_e)$ . Obviously, only  $z_1(kT_e)$  is available for control computation. Without loss of generality, suppose that system (6) is evolving on the right-hand side of phase plan, starting from initial conditions  $(z_{1,0}, z_{2,0})$  (point  $O$  - Figure 1-Left) which gives  $K(t) = K_m$  and  $u = -K_m$ . Then, system trajectories are parabola defined by

$$\begin{aligned} z_1(t) &= -K_m \frac{t^2}{2} + z_{2,0}t + z_{1,0}, \\ z_2(t) &= -K_m t + z_{2,0} \end{aligned} \quad (10)$$

Due to term  $\text{sign}(z_1(kT_e))$ , when system trajectories reach point  $A$ , control input sign does not change, whereas it changes when point  $B$  is reached<sup>2</sup> at

<sup>2</sup> Note that control input is switching with a delay w.r.t.  $z_1$  sign commutation which is a consequence of the finite sampling period  $T_e$ .

$t = t_B = k_B T_e$  ( $k_B \in \mathbb{N}$ ). From Theorem 1, one has  $t_B \in \mathcal{T}$  and, for  $t \in [t_B; t_B + T_e]$ ,  $u = K_M$ . It means that system trajectories follow

$$\begin{aligned} z_1(t) &= K_M \frac{(t - t_B)^2}{2} + z_2(t_B)(t - t_B) + z_1(t_B), \\ z_2(t) &= K_M(t - t_B) + z_2(t_B) \end{aligned} \quad (11)$$

for  $t \in [t_B; t_B + T_e]$ . Clearly, to reach a parabola closer to the origin than the symmetric of the parabola followed from  $O$  to  $B$  (dotted curve),  $K_M$  has to be sufficiently large. Then, point  $C$  is reached: from  $t_C = t_B + T_e$ , one applies  $u = K_m$ . The obtained system trajectory in the plan  $(z_1, z_2)$  is a parabola closer from the origin than the parabola containing points  $O$ ,  $A$  and  $B$ . When point  $E$  is reached at  $t_E = k_E T_e$ , control input equals, for  $t \in [t_E; t_E + T_e]$ ,  $u = -K_M$ .

**Remark 1** The gain commutation between  $K_m$  and  $K_M$  is necessary to achieve the convergence: without this commutation, system trajectories are diverging. ■

**Second Part.** Without loss of generality, suppose now that, at  $t = t_0$ ,  $[z_1 \ z_2]^T = [\epsilon_1 \ \epsilon_2]^T$ , with  $\epsilon_1, \epsilon_2$  positive constants. Suppose also that  $t_0 \in \mathcal{T}$  which gives  $\dot{z}_1 = z_2$ ,  $\dot{z}_2 = -K_M$  for  $t \in [t_0, t_0 + T_e]$ . Then, from  $t = t_0$  to  $t = t_0 + T_e$ , one has

$$\begin{aligned} z_2(t) &= -K_M \cdot (t - t_0) + \epsilon_2, \\ z_1(t) &= -K_M \cdot \frac{(t - t_0)^2}{2} + \epsilon_2 \cdot (t - t_0) + \epsilon_1 \end{aligned} \quad (12)$$

The maximum value for  $z_1(t)$  for  $t \in [t_0, t_0 + T_e]$ , denoted  $z_1^M$ , reads as  $z_1^M = \epsilon_1 + \epsilon_2^2 / K_M$ . Given that  $z_2(t_0) > 0$ , it is obvious that  $z_1(t_0 + T_e) > 0$ . From the first step of proof, the duration of parabolic trajectories decreases. Denoting  $t_1 = t_0 + T_e$ , define  $k_1 \in \mathbb{N}$  such that, for  $t \in [t_1, t_1 + (k_1 + 1) \cdot T_e]$ ,  $\dot{z}_1 = z_2$ ,  $\dot{z}_2 = -K_m$ , and

$$z_1(t_1 + k_1 \cdot T_e) > 0, \quad z_1(t_1 + (k_1 + 1) \cdot T_e) < 0.$$

Let  $\mathbf{L}_1$  denote the time interval length during which  $z_1$ -sign does not change: in the present case,  $\mathbf{L}_1 = (k_1 + 1) \cdot T_e$ . By solving the previous equations, one gets, with  $t_2 = t_1 + (k_1 + 1) \cdot T_e$ ,

$$\begin{aligned} z_2(t_2) &= -(K_m \cdot (k_1 + 1) + K_M) \cdot T_e + \epsilon_2 \\ z_1(t_2) &= -K_m \cdot \frac{(k_1 + 1)^2 \cdot T_e^2}{2} + \epsilon_2 \cdot T_e + \epsilon_1 + \\ &\quad (\epsilon_2 - K_M \cdot T_e) \cdot (k_1 + 1) \cdot T_e - K_M \cdot \frac{T_e^2}{2} \end{aligned} \quad (13)$$

Denoting  $\Delta z_2$  the difference between maximal and minimal values of  $z_2(t)$ , it is obvious that

$$\Delta z_2 = z_2(t_0) - z_2(t_2) = (K_m \cdot (k + 1) + K_M) \cdot T_e \quad (14)$$

For  $t \in [t_2, t_2 + T_e]$ , one has  $\dot{z}_1 = z_2$  and  $\dot{z}_2 = K_M$ . The minimum value of  $z_1$  during this time interval denoted  $z_1^m$  is derived from  $dz_1/dt = 0$ . From first step of proof, it is clear that  $|z_1^m| \leq |z_1^M|$ . It yields that  $\mathbf{L_I}$  is decreasing to  $\mathbf{L_I} = T_e$ .

The objective is now to prove that trajectories converge to a stable periodical cycle. When  $\mathbf{L_I} = T_e$ , at each sampling period  $T_e$ , system trajectories are changing of phase plan part in the clockwise which means that  $\dot{z}_2$  takes the successive values  $+K_M$ ,  $+K_M$ ,  $-K_M$  and  $-K_M$ . Then, the average value of  $\dot{z}_2$  over the four sampling periods equals 0. It also means that  $z_2$  is periodical, as  $z_1$ . Furthermore, when  $\mathbf{L_I} = T_e$ , it can be proved that the limit cycle width in  $z_1$ -axis equals

$$\Delta z_1 = \frac{K_M - K_m}{2} \cdot T_e^2 + \frac{(K_M + K_m)^2}{4K_M} \cdot T_e^2$$

which gives first condition of (9). As  $z_1$  is periodical and allows a constant magnitude  $\Delta z_1$ , it means that  $z_2$  allows a null average value. Then, from  $\Delta z_2$ , one gets a *stable limit cycle* such that

$$\text{Max}(z_2) = -\text{min}(z_2) = \frac{K_m + K_M}{2} \cdot T_e$$

which gives the second condition of (9). ■

### 3.2 Control of uncertain nonlinear system

Output feedback controller is now proposed for continuous uncertain nonlinear systems.

**Theorem 2** Consider nonlinear system (1) with sliding variable  $s(x, t)$  defined by (2). Suppose that assumptions H1, H2 and H3 are fulfilled, and state the gain  $K_m$  such that  $K_m > a_M/b_m$ . Then, there always exists a sufficiently large gain  $K_M$  with  $0 < K_m < K_M < \infty$  such that the gain  $K(t)$  defined as

$$K(t) = \begin{cases} K_m & \text{if } t \notin \mathcal{T} \\ K_M & \text{if } t \in \mathcal{T} \end{cases} \quad (15)$$

with  $\mathcal{T} = \{t \mid \text{sign}(s(kT_e)) \neq \text{sign}(s((k-1)T_e))\}$ ,  $k \in \mathbb{N}$  and the control input

$$u = -K(t) \cdot \text{sign}(s(kT_e)) \quad (16)$$

ensure the establishment of a real second order sliding mode for system (1) with respect to sliding variable  $s(x, t)$ . ■

**Proof.** As previously, the proof is composed by 2 steps.

**First step.** Without loss of generality, suppose that system (3) is starting from initial conditions

$(z_{1,0}, z_{2,0})$  (point  $O$  - Figure 1-Right) which gives  $K(t) = K_m$  and  $u = -K_m$ . One gets  $\dot{z}_1 = z_2$  and  $\dot{z}_2 = a - K_m b$ . Then, one has  $z_{1m}(t) < z_1(t) < z_{1M}(t)$  with  $z_{1m}(t) = -(K_m b_M + a_M) \frac{t^2}{2} + z_{2,0}t + z_{1,0}$ ,  $z_{1M}(t) = -(K_m b_m - a_M) \frac{t^2}{2} + z_{2,0}t + z_{1,0}$  and

$$-(K_m b_M + a_M)t + z_{2,0} < z_2 < -(K_m b_m - a_M)t + z_{2,0}.$$

It means that system is evolving in a domain defined by upper- and lower- parabolas (see domain between dotted lines  $(OB_1)$  and  $(OB_2)$  - Figure 1-Right). As in the previous “ideal” case, from Theorem 2, one has  $t_B \in \mathcal{T}$ : then, for  $t \in [t_B; t_B + T_e]$ ,  $u = K_M$  and

$$\dot{z}_1 = z_2, \quad \dot{z}_2 = a + K_M b \quad (17)$$

Thanks to an adequate gain  $K_M$ , system trajectories reach trajectories family closer to the origin than the previous one (Point  $C$  - Figure 1-Right). This trajectories family is limited by upper- and -lower parabolas (see domain between dotted lines  $(OC_1)$  and  $(OC_2)$  - Figure 1-Right). Then, gain  $K_M$  has to be sufficiently large. Point  $E$  being reached at  $t_E = k_E T_e$ , one gets, for  $t \in [t_E; t_E + T_e]$ ,  $u = -K_M$ . Note that

- If  $u = K_m$  for  $t \in \mathcal{T}$ , system follows trajectories family which could engender divergence (see point  $F$  and trajectories family limited by  $(BD_1)$  and  $(BD_2)$ ).
- As shown by Figure 1-Right, the point  $C$  is located in a trajectories family closer to the origin.  $K_M$  must be sufficiently large so that system reaches this domain.

**Second step.** The idea here consists in using the result of “ideal” double integrator in order to prove that, in uncertain case, system is reaching a vicinity of the origin whose limits depend on  $T_e$  and  $T_e^2$ . The previous proof methodology can be used in the uncertain case by supposing the “worst” case, *i.e.* the trajectories resulting from the maximum values of uncertainties. In the “worst” case, these trajectories are evolving between two parabolas. Then, it can be derived that  $|s| \leq k_1 T_e^2$  and  $|\dot{s}| \leq k_2 T_e$  with  $k_1, k_2$  positive constants. ■

## 4 An academic example

The system (Levant, 2007) (Figure 2) is a variable-length pendulum evolving in a vertical plane is displayed in the sequel. In (Levant, 2007), performances of second order sliding mode controller with sliding mode differentiators are evaluated on this system; in (Plestan *et al.*, 2008b), a first attempt for second order sliding mode output feedback control is proposed. The sampling time (control computation period) is larger than integration steps (not the case in (Levant, 2007)). Thus, simulations have been made with a control input sampling time at least 100 times higher than the integration step ( $10^{-5}$  sec) in order to well-simulate the continuous plant. Furthermore,

the control solution *only* needs pendulum angular position whereas, in (Levant, 2007), both angular position and velocity are needed, and in (Plestan *et al.*, 2008b), the sign of its angular velocity is also needed. System dynamics reads as

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2\frac{\dot{R}(t)}{R(t)}x_2 - \frac{g}{R(t)}\sin(x_1) + \frac{1}{mR(t)^2}u\end{aligned}\quad (18)$$

with  $(x_1, x_2)$  the angular position and velocity of the rod,  $m = 1$  kg the load mass,  $g = 9.81$  ms<sup>-2</sup> the gravitational constant,  $R(t)$  the distance from the fix point  $O$  and the mass, and  $u$  the control torque.  $R(t)$  is a non-measured disturbance and reads as  $R(t) = 0.8 + 0.1\sin(8t) + 0.3\cos(4t)$ . Function  $R(t)$  and its time derivative  $\dot{R}(t)$  are such that  $0.4515 \leq R(t) \leq 1.1485$  and  $-2.5226 \leq \frac{\dot{R}(t)}{R(t)} \leq 1.4989$ . With  $s(x, t) = x_1 - 0.5\sin(0.5t) - 0.5\cos(t)$ , system is initialized such that  $s(0) = -0.5$  rad and  $\dot{s}(0) = -0.25$  rad · s<sup>-1</sup>. One has

$$\ddot{s} = \underbrace{\left[-\frac{2\dot{R}(t)}{R(t)}x_2 - \frac{g}{R(t)}\sin(x_1)\right]}_{=a(x,t)} + \underbrace{\left[\frac{1}{mR(t)^2}\right]}_{=b(t)}u.$$

For  $x_2 \in [-10, 10]$ , one gets  $|a(t)| < 72.18$  and  $0 < 0.7581 \leq b(t) \leq 4.9055$ . As gain  $K_m$  has to fulfill  $K_m > a_M/b_m$ , one gets  $K_m > 95.21$ . It is stated as  $K_m = 150$ . The control sampling period is stated as  $T_e = 0.001$ s, whereas the choice for  $K_M$  is  $K_M = 800$ . It implies that system trajectories converge, in a finite time, in the neighborhood of desired trajectories (Figure 3). Controller performances are at least so good than (Levant, 2007), knowing that *only*  $x_1$  is used whereas  $x_2$  is required in (Levant, 2007). Precision on  $s$  and  $\dot{s}$  is linked to  $T_e^2$  and  $T_e$  respectively (see Table 1): a “real” second order sliding mode is well-established.

$T_e$ (sec)	$10^{-3}$	$10^{-4}$
Max( $ s $ ) (rad)	$3.1 \cdot 10^{-2}$	$4 \cdot 10^{-4}$
Max( $ \dot{s} $ ) (rad.s <sup>-1</sup> )	7.1	0.78

Table 1  
Evaluation of the maximum values of  $|s|$  and  $|\dot{s}|$  in case of two different  $T_e$  values for  $t \in [6 \text{ sec}; 12 \text{ sec}]$ .

## 5 Conclusion

A new strategy for second order sliding mode control based on output feedback for a large class of uncertain sampling controlled systems is proposed. This control law has been established by taking into account the sampling time period  $T_e$ . Only the sliding variable information is required. Futurs works on this topic will concern

experimentations and use of this control strategy for robust observers design.

## References

- Anosov, D.V. (1959). On stability of equilibrium points of relay systems. *Automatica i Telemekhanika (Automation and Remote Control)* **2**, 135–149.
- Bartolini, G., A. Ferrara, E. Usai and V.I. Utkin (2000). On multi-input chattering-free second-order sliding mode control. *IEEE Trans. Autom. Control* **45**, 1711–1717.
- Filippov, A.F. (1988). *Differential equations with discontinuous right-hand side*. Kluwer. Dordrecht.
- Fridman, L. and A. Levant (1996). Sliding modes of higher order as a natural phenomenon in control theory. In: *Robust Control via Variable Structure and Lyapunov Techniques, Lecture Notes in Control and Information Sciences*. Vol. 217. Springer Verlag, Ed. F. Garofalo, L. Glielmo.
- Fridman, L. and A. Levant (2002). Higher order sliding modes. In: *Sliding Mode Control in Engineering*. Marcel Dekker, Ed. J.P. Barbot, W. Perruquetti. pp. 53–101.
- Girin, A., F. Plestan, X. Brun and A. Glumineau (2009). Robust control of an electropneumatic actuator : application to an aeronautical benchmark. *IEEE Trans. Control Syst. Technology* **17**(3), 633–646.
- Khan, M.K., S.K. Spurgeon and A. Levant (2003). Simple output-feedback 2-sliding controller for systems of relative degree two. In: *Proc. European Control Conference ECC'03*. Cambridge, UK.
- Laghrouche, S., F. Plestan and A. Glumineau (2006a). Practical higher order sliding mode control: optimal control based approach and application to electromechanical systems. In: *Advances in Variable Structure and Sliding Mode Control, Lecture Notes in Control and Information Sciences*. Vol. 334. Springer-Verlag, Ed. C. Edwards, E. Fossas Colet, L. Fridman.
- Laghrouche, S., F. Plestan and A. Glumineau (2007). Higher order sliding mode control based on integral sliding surface. *Automatica* **43**(3), 531–537.
- Laghrouche, S., M. Smaoui, F. Plestan and X. Brun (2006b). Higher order sliding mode control based on optimal approach of an electropneumatic actuator. *International Journal of Control* **79**, 119–131.
- Levant, A. (1993). Sliding order and sliding accuracy in sliding mode control. *International Journal of Control* **58**, 1247–1263.
- Levant, A. (2001). Universal siso sliding-mode controllers with finite-time convergence. *IEEE Trans. Autom. Control* **49**, 1447–1451.
- Levant, A. (2003). Higher-order sliding modes, differentiation and output-feedback control. *International Journal of Control* **76**, 924–941.
- Levant, A. (2005). Homogeneity approach to high-order sliding mode design. *Automatica* **41**, 823–830.
- Levant, A. (2007). Principles of 2-sliding mode design. *Automatica* **43**(4), 576–586.
- Moulay, E. and W. Perruquetti (2005). Finite time stability of differential inclusions. *IMA Journal of Mathematical Control and Information* **22**(4), 465–475.

Moulay, E. and W. Perruquetti (2006). Finite-time stability and stabilization: state of the art. In: *Advances in Variable Structure and Sliding Mode Control, Lecture Notes in Control and Information Sciences*. Vol. 334. Springer-Verlag, Ed. C. Edwards, E. Fossas Colet, L. Fridman. pp. 23–41.

Plestan, F., A. Glumineau and S. Laghrouche (2008a). A new algorithm for high order sliding mode control. *International Journal of Robust and Nonlinear Control* **18**(4–5), 441–453.

Plestan, F., E. Moulay, A. Glumineau and T. Cheviron (2008b). Second order sliding mode control: a new output feedback solution. In: *Proc. IFAC World Congress*. Seoul, Korea.

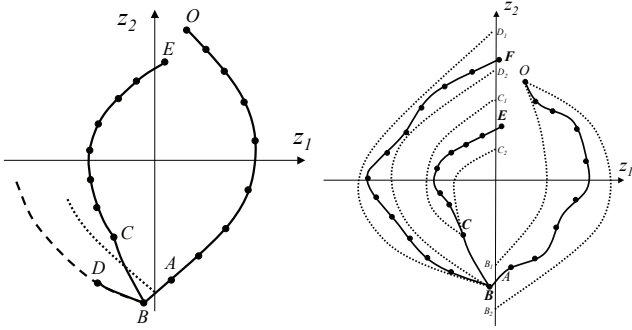


Figure 1. **Left-Double integrator.** Phase portrait of system (6). **Right-Uncertain system.** Phase portrait of system (3)

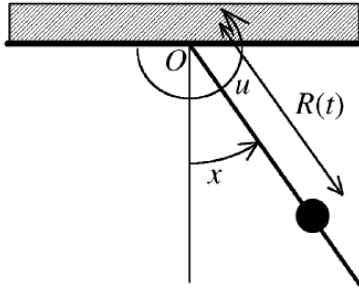


Figure 2. Pendulum scheme.

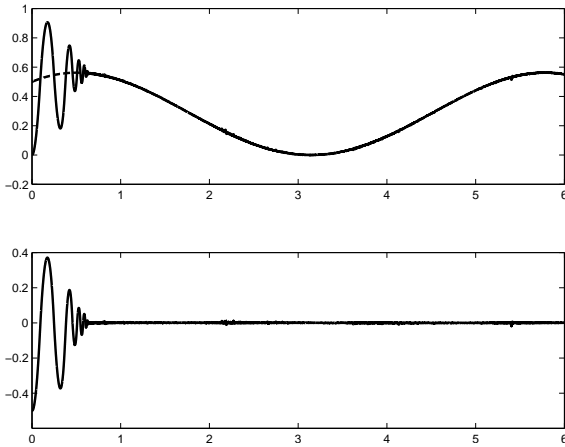


Figure 3. Angular position  $x_1$  (rad - solid line) and its reference trajectory (dashed line) versus time (sec).